PG-CS-1114	MMSS-11

P.G. DEGREE EXAMINATION – FEBRUARY 2023.

Mathematics

First Semester

ABSTRACT ALGEBRA

Time : 3 hours Maximum marks : 70

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE of the following.

1. Let G be a group and let p be a prime. If $o(G) = p^2$, then show that G is abelian.

2. State and prove the division algorithm for polynomial rings.

3. Define a primitive polynomial. Show that product of two primitive polynomials is also a primitive polynomial.

4. For any $f(x), g(x) \in F[x]$, show that $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$.

5. Show that for every prime p and every positive integer m there exists a field having p^m elements.

6. If A, B are finite subsets of a group G, then show that $0(AxB) = \frac{o(A)o(B)}{o(A \cap xBx^{-1})}$.

7. Define internal direct product of subgroups and external direct product of groups.

8. State and prove the Gauss' lemma.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE of the following.

9. State and prove first part of Sylow's theorem.

10. Show that every finite abelian group is the direct product of cyclic groups.

11. If L is a finite extension of K and if K is a finite extension of F, then show that L is a finite extension of F. Also, show that

$[L:F] = [L:K][K:F]$.

12. State and prove the fundamental theorem of Galois theory.

13. State and prove the Wedderbun's theorem on finite division rings.

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PG–CS-1115	MMSS–12

P.G. DEGREE EXAMINATION — FEBRUARY 2023.

Mathematics

First Semester

ADVANCED CALCULUS

Time : 3 hours Maximum marks : 70

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE of the following.

1. If $u + \log u = xy$, $u + \log v = x - y$, find $\frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial x}$.

2. Verify whether the functions $u = \frac{x+y}{1-xy}$, $v = \tan^{-1} x + \tan^{-1} y$ are functionally dependent, and if so, find the relation between them.

3. Find the shortest distance from (1,0) to the parabola $y^2 = 4x$.

4. If $X(x,y) = 2x^2 y, Y(x,y) = 3xy$, find the work done by the field on a particle moving from (0,0) to (1,4) along the curve $y = 4x^2$.

5. Evaluate by Stroke's theorem the integral $\int_S x^2 z dx + xy dy$, where S is the rectangle in the plane z = 0, where the sides are along the lines $x = 0, y = 0, x = a, y = b$.

6. Find directional derivative of $f(x,y) = x^2 - 2y$ in the direction $\xi_{\frac{3\pi}{4}}$ at (1, 2).

7. Expand $f(x,y) = x^3 - 2xy^2$ in Taylor's series at (1, –1).

8. Find the area enclosed by the folium $x^3 + y^3 = 3axy$.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE of the following.

9. State and prove Euler's theorem and its converse.

10. State and prove the existence theorem for implicit functions.

11. If $f(x,y) \in C^2, f_1 = f_2 = 0$ at (X,Y) and if $f_{12}^2 - f_{11}f_{22} > 0$ at (X, Y), then show that $f(x,y)$ has a saddle point at (X,Y).

12. Verify Gauss theorem for $\iint_S (4x\cos\alpha - 2y^2\cos\beta + z^2\cos\gamma)\,dS$, where S is the region bounded by $x^2 + y^2 = 4, z = 0, z = 3$ and α, β, γ are the angle between the exterior normal to the positive x-axis, y-axis and z-axis respectively.

13. State and prove Stroke's theorem.

PG-CS-1116	**MMSS-13**

P.G. DEGREE EXAMINATION – FEBRUARY 2023.

Mathematics

First Semester

REAL ANALYSIS

Time : 3 hours   Maximum marks : 70

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE of the following.

1. Show that $\int_{\underline{a}}^{b} f\,d\alpha \le \int_{a}^{\overline{b}} f\,d\alpha$.

2. Does convergent series of continuous functions converge to a discontinuous function? Justify.

3. Show that there exists a non-measurable set.

4. If $a > 1$, show that $\int_0^1 \frac{x \sin x}{1+(nx)^{\alpha}}\,dx = o\left(n^{-1}\right)$ as $n \to \infty$.

5. Show that the following are equivalent for a linear functional G:

   (a) G is bounded

   (b) G is continuous at 0.

   (c) G is continuous at each $x \in V$.

6. If f is a continuous mapping of a metric space X into a metric space Y. and if E is a connected subset of X, then show that $f(E)$ is connected.

7. Suppose $\{f_n\}$ is a sequence of functions defined on E, and suppose that $|f_n(x)| \leq M, \quad (x \in E, n = 1, 2, ...)$. Then show that $\sum f_n$ converges uniformly on E if $\sum M_n$ converges.

8. Show that if f is a non-negative measurable function, then $f = 0$ *a. e.* if and only if $\int f\, dx = 0$.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE of the following.

9. Assume α increases monotonically and $\alpha' \in \mathcal{R}$ on $[a, b]$. Let f be a bounded real function on $[a, b]$. Show that $f \in \mathcal{R}(\alpha)$ if and only if $f\alpha' \in \mathcal{R}$. Also show that

$$\int_a^b f\, d\alpha = \int_a^b f(x)\alpha'(x)dx .$$

10. State and prove the Stone-Weierstrass theorem.

11. Show that the outer measure of an interval equals its length.

12. State and prove the Fatou's lemma.

13. State and Riesz representation for L_p, $p > 1$.

PG–CS-1117	**MMSSE–1**

P.G. DEGREE EXAMINATION — FEBRUARY 2023.

Mathematics

First Semester

DIFFERENTIAL GEOMETRY

Time : 3 hours — Maximum marks : 70

PART A — (5 × 5 = 25 marks)

Answer any FIVE Questions.

All questions carries equal marks.

1. Prove that the radius of curvature of the locus of the centre of curvature of a curve is given by $\left[\left\{\frac{\rho^2\sigma}{R^3}\frac{d}{ds}\left(\frac{\sigma\rho'}{\rho}\right)-\frac{1}{R}\right\}^2+\frac{\rho'^2\sigma^4}{\rho^2R^4}\right]^{-\frac{1}{2}}$.

2. Prove that the metric is invariant under a parametric transformation.

3. Prove that the curve of the family $\frac{v^3}{u^2}=$ constant are geodesics on a surface with metric $v^2du^2-2uvdudv+2u^2dv^2\left(u>0, v>0\right)$.

4. Prove that, if there is a surface of minimum are passing through a closed space curve, it is necessarily a minimal surface.

5. State and prove Hilbert's lemma.

6. Show that the torsion of an involute of a curve is equal to $\frac{\rho(\sigma\rho' - \sigma'\rho)}{(\rho^2+\sigma^2)(c-s)}$.

7. State and prove Liouville's formula theorem.

8. Prove that the principal directions are given by $(EM-FL)l^2 + (EN-GL)lm + (FN-GM)m^2 = 0$.

PART B — (3 × 15 = 45 marks)

Answer any THREE questions.

All questions carries equal marks.

9. Obtain the curvature and to torsion of the curve of intersection of the two quadric surfaces $ax^2 + by^2 + cz^2 = 1$; $a'^{x^2} + b'y^2 + c'z^2 = 1$.

10. If (l, m) and (l', m') are the direction coefficients of two directions at a point P on the surface and θ is the angle between the two direction at P, then prove that

    (a) $\cos\theta = Ell' + F(lm' + l'm)Gmm'$

    (b) $\sin\theta = H(lm' - l'm)$

11. State and prove Gauss-Bonnet theorem.

12. Prove that a necessary and sufficient condition for a surface to be a developable is that its Gaussian curvature shall be zero.

13. Prove that, when the surface S has negative curvature everywhere, the length of a geodesic which joins any two points A, B is always less than the lengths of neighbouring curves through A and B.

PG–CS–1118	MMSSE–2

P.G. DEGREE EXAMINATION, FEBRUARY 2023

Mathematics

First Semester

PROGRAMMING IN C++

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Explain the characteristics of OOPs.
2. Write a C++ program to sort the given numbers using function.
3. Explain array of objects with an example.
4. What is destructor? Illustrate with an example.
5. Write note on Virtual functions with example program.
6. Explain the Friend function concept with an example program.
7. Write a program in C++ to check whether the given no is prime or not.
8. Explain function prototyping with example.

PART B — (3 × 15 = 45 marks)

Answer any THREE questions.

9. Explain the various operators that are available in C++ with neat illustration for each it.

10. Write a C++ program to find the area of the square, rectangle, circle using function overloading.

11. What is Constructor? Explain types of Constructor with example.

12. Define operator overloading? Explain how to overload unary operator and binary operator.

13. What does inheritance means in C++? What are different forms of inheritance? Give an example of each.

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PG–CS–1119	MMSS–21

P.G. DEGREE EXAMINATION — FEBRUARY 2023

Mathematics

Second Semester

APPLIED MECHANICS

Time : 3 hours Maximum marks : 70

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

All questions carries equal marks.

1. Find the components of velocity and acceleration in cyclindrical coordinates.

2. Find the equation of the motion in cylindrical coordinates.

3. Discuss the steady precession of a top.

4. Consider a spherical pendulum consisting of a particle of mass m which moves under gravity on a smooth sphere of radius a. In terms of spherical polar angles θ, ϕ with θ measured up from the downward vertical. Find the equation motion of a sperical pendulam.

5. If $\left(q^*, p^*\right)$ is the position in phase space at time t^* of a particle of the dynamical fluid and (q, p) the position of the same partical at time t, the prove that the transformation $\left(q^*, p^*\right) \rightarrow (q, p)$ is a canonical transformation and Hamilton's characteristic function is a generating function of that transformation.

6. Explain Generating functions.

7. Explain the general method of finding principal axes and Moments of Inertia.

8. Explain principal of angular momentum.

PART B — $(3 \times 15 = 45$ marks)

Answer any THREE questions.

All questions carries equal marks.

9. Find the angular momentum of a particle and of a system of particles.

10. Explain the motion of a rigid body with a fixed point.

11. Explain the motion of a rigid body with a fixed point under no forces using descriptive method.

12. Derive Lagrange's equations for a particle in a plane.

13. State and Prove Liouville's theorem.

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PG-CS-1120	MMSS–22

M.Sc. DEGREE EXAMINATION – FEBRUARY, 2023.

Mathematics

Second Semester

COMPLEX ANALYSIS

Time : 3 hours | Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

All questions carries equal marks.

1. State and prove that the Weierstrass theorem on essential singularity.

2. State and prove Residue theorem.

3. Derive Legendre's duplication formula.

4. Prove that a continuous function $u(z)$ which satisfies the mean-value property is necessarily Harmonic.

5. Show that any two basis of the same module are connected by a unimodular transformation.

6. State and prove mean value property.

7. Prove that a family $\mathfrak{I}$ of analytic functions is normal with respect to $\mathbb{C}$ if and only if the functions in $\mathfrak{I}$ are uniformly bounded on every compact set.

8. State and prove Cauchy's theorem in disk.

PART B — (3 × 15 = 45 marks)

Answer any THREE questions.

All questions carries equal marks.

9. If the piecewise differentiable closed curve γ does not pass through the point a, then prove that the value of the integral $\int_{\gamma} \frac{dz}{z-a}$ is a multiple of $2\pi i$.

10. Derive Possison's formula.

11. State and prove Hadamard's theorem.

12. State and prove Schwarz-Christoffel formula theorem.

13. Obtain the first order differential equation satisfied by $\wp(z)$.

PG–CS–1121	**MMSS–23**

P.G. DEGREE EXAMINATION —
FEBRUARY, 2023.

Mathematics

Second Semester

LINEAR ALGEBRA

Time : 3 hours Maximum marks : 70

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.
All questions carries equal marks

1. Let V and W be vector spaces over the field F and let T be a linear transformation from V into W. If T is invertible, then prove that the inverse function T^{-1} is a linear transformation from W onto V.

2. If *f, d* are polynomials over a field F and d is different from 0 then prove that there exist polynomials *q,r* in *F[X]* such that *(i) f = dq + r (ii) either r* 0 *or deg r* < deg d. The polynomials q, *r* satisfying (i) and (ii) are unique.

3. Let K be a commutative ring with identity. and let A and B be n X n matrices over K. Then prove that det (AB) = (det A) (det B)

4. Let V be a finite-dimensional vector space over the field F and let T be a linear operator on V. Then prove that T is triangulable if and only if the minimal polynomial for T is a product of linear polynomials over F.

5. State and prove Generalised Cayley-Hamilton theorem.

6. Prove that every n-dimensional vector space over the field F is isomorphic to the space F^n.

7. If F is a field and M is any non-zero ideal in *F[x],* then prove that there is a unique monic polynomial d in *F* [*x*] such that M is the principal ideal generated by d.

8. Let V be a finite-dimensional vector space. Let W_1, ..., W_k be subspaces of V and let $W = W_1 + \ldots + Wk$. Then prove that the following are equivalent

   (a) W_1, ..., W_k are independent.

   (b) for each $j, 2 \leq j \leq k, w_1 \cap (w_1 + \ldots + w_k) = \{0\}$

   (c) If $\mathbb{B}_1$ is an ordered basis for $W_i, 1 \leq i \leq k,$. then the sequence $\mathbb{B} = (\mathbb{B}_1, \ldots, \mathbb{B}_k)$ is an ordered basis for W.

PART B — (3 × 15 = 45 marks)

Answer any THREE questions.
All questions carries equal marks

9. (a) Let T be a linear transformation from V into W. Then prove that T is non-singular if and only if T carries each linearly independent subset of V onto a linearly independent subset of W.

   (b) If A is an m x n matrix with entries in the filed F, then prove that row rank(A) = column rank (A)

10. Prove that, if F is field, a non-scalar monic polynomial in $F\,[x]$ can be factored as a product of monic primes in $F[x]$ in one and, expect for order, only one way.

11. State and prove Cayley-Hamilton theorem.

12. State and prove Primary Decomposition theorem.

13. Let A be the complex matrix

$$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 \\ -1 & 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

Find the Jordan form for A.

PG–CS–1123	**MMSSE–4**

P.G. DEGREE EXAMINATION — FEBRUARY 2023

Mathematics

Second Semester

MATHEMATICAL STATISTICS

Time : 3 hours Maximum marks : 70

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE of the following.

1. Define an unbiased estimator of the parameter θ. If X has the binomial distribution with the parameters n and θ, show that the sample proportion, $\frac{x}{n}$, is an unbiased estimator of θ.

2. If $x = 4$ of $n = 20$ patients suffered serious side effects from a new medication, test the null hypothesis $\theta = 0.50$ against the alternative hypothesis $\theta \neq 0.50$ at the 0.05 level of significance. Here θ is the true proportion of patients suffering serious side effects from the new medication.

3. Given two random variables X and Y that have the joint density

$$f(x,y)=\begin{cases} x\,e^{-x(1+y)}, & \text{for } x>0 \text{ and } y>0 \\ 0, & \textit{elsewhere} \end{cases}$$

Find the regression equation of Y on X.

4. Write a short note on Latin square design of experiment.

5. Compute the correlation matrix ρ from the covariance matrix $\begin{bmatrix} 4 & 1 & 2 \\ 1 & 9 & -3 \\ 2 & -3 & 25 \end{bmatrix}$.

6. Show that $Y=\frac{1}{6}(X_1+2X_2+3X_3)$ is not a sufficient estimator of the Bernoulli parameter θ.

7. Write a short note on Type I and Type II errors.

8. Show that the sum of the products of any variable with every residual is zero, provided the subscript of the variable occurs among the secondary subscripts of the residual.

SECTION B — ($3 \times 15 = 45$ marks)

Answer any THREE of the following.

9. (a) If $X_1, X_2, \ldots, X_n$ constitute a random sample of size n from a normal population with the mean μ and variance σ^2, find the maximum likelihood estimates of these two parameters.

(b) In a random sample of 136 of 400 persons given flu vaccine experienced some discomfort. Construct a 95% confidence interval for the true proportion of persons who will experience some discomfort from the vaccine.

10. State and prove the Neyman-Pearson Lemma.

11. Prove that the necessary and sufficient condition for the three regression planes to coincide is $r_{12}^2 + r_{23}^2 + r_{31}^2 - 2r_{12}r_{23}r_{31} = 1$.

12. A completely randomized design experiment with 10 plots and 3 treatments gave the following results.

Plot. No.	1	2	3	4	5	6	7	8	9	10
Treatment	A	B	C	A	C	C	A	B	A	B
Yield	5	4	2	7	5	1	3	4	1	7

Analyze the results for treatment effects.

13. Evaluate the $\rho = 2$-variate normal density in terms of the individual parameters $\mu_1 = E(X_1)$, $\mu_2 = E(X_2)$. $\sigma_{11} = Var(X_1)$, $\sigma_{22} = Var(X_2)$ and $\rho_{12} = \frac{\sigma_{12}}{\sqrt{\sigma_{11}}\sqrt{\sigma_{22}}} = Corr(X_1, X_2)$.

PG-CS-1128	MMSSE-5

P.G. DEGREE EXAMINATION – FEBRUARY 2023.

Mathematics

Third Semester

GRAPH THEORY

Time : 3 hours Maximum marks : 70

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions out of Eight questions.

All questions carry equal marks.

1. Define the following of a graph G:

   (a) Radius

   (b) Diameter

   (c) Girth.

2. Prove that there are exactly two isomorphism classes 4-regular simple graphs with 7 vertices.

3. Prove that if G is Hamiltonian then for every non-empty proper subset S of V, $\omega(G - S) \le |S|$.

4. Apply Mycielskian construction for 5-cycle graph and sketch the resulting graph.

5. Prove that K_5 is non-planar.

6. Find the chromatic polynomial of K_4.

7. Briefly write about use of Wang and Kleitman's algorithm.

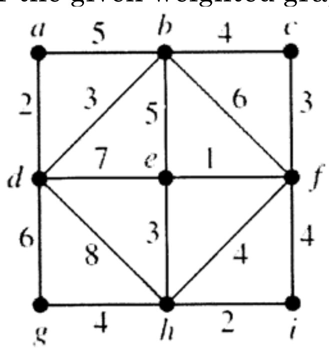
8. Define the following
   (a) Matching
   (b) Perfect Matching
   (c) Maximum matching in a graph G.

PART B — (3 × 15 = 45 marks)

Answer any THREE questions out of Five questions

All questions carry equal marks.

9. Use Prim's algorithm to find a minimum spanning tree (MST) for the given weighted graph.



10. State and Prove Menger's theorem.

11. Prove that a simple graph G is Eulerian if and only if it is connected and every vertex has an even degree.

12. State and prove Vizing's theorem.

13. State and prove Euler's formula on planarity.